Direct CP Violation and Isospin Triangles of $B \to \pi\pi$ Decays

Zhi-zhong Xing

CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China
and Institute of High Energy Physics, Chinese Academy of Sciences,
P.O. Box 918 (4), Beijing 100039, China *
(Electronic address: xingzz@mail.ihep.ac.cn)

Abstract

The recent observation of $B^0_d \to \pi^0\pi^0$ and $\bar{B}^0_d \to \pi^0\pi^0$ decay modes allows us to make a fresh isospin analysis of $B \to \pi\pi$ transitions. We find that current experimental data can impose some model-independent constraints on the parameter space of direct CP violation in $B_d \to \pi^+\pi^-$ and $B_d \to \pi^0\pi^0$. Furthermore, we establish a direct relationship between the weak phase α and the charge-averaged branching fractions and CP-violating asymmetries of $B \to \pi\pi$ decays.

PACS number(s): 12.15.Ff, 13.25.Hw, 11.30.Er, 14.40.Nd

*Mailing address

The major goal of B-meson factories is to test the Kobayashi-Maskawa mechanism of CP violation within the standard model and to detect possible new sources of CP violation beyond the standard model. So far the CP-violating asymmetry in B^0_d vs $\bar{B}^0_d \rightarrow J/\psi K_{\rm S}$ decays has unambiguously been measured at KEK and SLAC [1], and the experimental result is compatible very well with the standard-model expectation. Some preliminary evidence for CP violation in a number of other B decays has also been reported by BaBar and Belle Collaborations [2]. Further experiments will provide much more precise data on CP violation in the B-meson system, from which one may cross-check the consistency of the Kobayashi-Maskawa picture and probe possible new physics.

The charmless two-body nonleptonic decays $\bar{B}^0_d \rightarrow \pi^+\pi^-$, $\bar{B}^0_d \rightarrow \pi^0\pi^0$ and $B^-_u \rightarrow \pi^0\pi^-$, which can be related to one another via the isospin triangle [3], have been of great interest in B physics for a stringent test of the factorization hypothesis, a quantitative analysis of final-state interactions, and a clean determination of the CP-violating phase $\alpha \equiv \arg[-(V_{td}V^*_{tb})/(V_{ud}V^*_{ub})]$ (an inner angle of the well-known unitarity triangle of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [4]). Among these three decay modes, $\bar{B}^0_d \rightarrow \pi^0\pi^0$ is color-suppressed and should have the smallest branching fraction. The observation of this rare decay has recently been reported by BaBar [5] and Belle [6] Collaborations [1]:

$$
\begin{aligned}
\mathcal{B}_{00} &\equiv \frac{\mathcal{B}(B^0_d \rightarrow \pi^0\pi^0) + \mathcal{B}(\bar{B}^0_d \rightarrow \pi^0\pi^0)}{2} \\
&= \left\{ \begin{array}{ll} (2.1 \pm 0.6 \pm 0.3) \times 10^{-6} & \text{(BaBar)} \\ (1.7 \pm 0.6 \pm 0.3) \times 10^{-6} & \text{(Belle)}\,, \end{array} \right.
\end{aligned}
\tag{1}
$$

where the first error is statistical and the second one is systematic. The experimental value of $\mathcal{B}_{00}$ is a bit larger than the theoretical prediction in various QCD models [7]. In comparison, the observed branching fractions of $\bar{B}^0_d \rightarrow \pi^+\pi^-$ and $B^-_u \rightarrow \pi^0\pi^-$ decays [8]

$$
\begin{aligned}
\mathcal{B}_{+-} &\equiv \frac{\mathcal{B}(B^0_d \rightarrow \pi^+\pi^-) + \mathcal{B}(\bar{B}^0_d \rightarrow \pi^+\pi^-)}{2} \\
&= \left\{ \begin{array}{ll} (4.7 \pm 0.6 \pm 0.2) \times 10^{-6} & \text{(BaBar)} \\ (4.4 \pm 0.6 \pm 0.3) \times 10^{-6} & \text{(Belle)} \\ (4.5 \pm 1.4 \pm 0.5) \times 10^{-6} & \text{(CLEO)} \end{array} \right.
\end{aligned}
\tag{2}
$$

and

$$
\begin{aligned}
\mathcal{B}_{0\pm} &\equiv \frac{\mathcal{B}(B^+_u \rightarrow \pi^0\pi^+) + \mathcal{B}(B^-_u \rightarrow \pi^0\pi^-)}{2} \\
&= \left\{ \begin{array}{ll} (5.5 \pm 1.0 \pm 0.6) \times 10^{-6} & \text{(BaBar)} \\ (5.3 \pm 1.3 \pm 0.5) \times 10^{-6} & \text{(Belle)} \\ (4.6 \pm 1.8 \pm 0.7) \times 10^{-6} & \text{(CLEO)} \end{array} \right.
\end{aligned}
\tag{3}
$$

are essentially consistent with the theoretical results [7]. Because $\mathcal{B}_{+-}$, $\mathcal{B}_{00}$ and $\mathcal{B}_{0\pm}$ are all charge-averaged, they cannot directly be related to one another through the isospin relation.

[1] Note that the experimentally-reported branching fractions of $B \rightarrow \pi\pi$ decays are all charge-averaged.

The purpose of this note is to make a fresh isospin analysis of three $B \to \pi\pi$ decays by using new experimental data and taking account of CP violation. We find that current data can impose some useful and model-independent constraints on the parameter space of direct CP violation in $B_d \to \pi^+\pi^-$ and $B_d \to \pi^0\pi^0$ transitions. It is also possible to establish a direct relationship between the weak phase α and the charge-averaged branching fractions and CP-violating asymmetries of $B \to \pi\pi$ decays.

Let us define the *direct* CP-violating asymmetries between $\bar{B}^0_d \to \pi^+\pi^-$, $\bar{B}^0_d \to \pi^0\pi^0$, $B^-_u \to \pi^0\pi^-$ and their CP-conjugate decays:

$$\begin{aligned} \mathcal{A}_{+-} &\equiv \frac{\mathcal{B}(\bar{B}^0_d \to \pi^+\pi^-) - \mathcal{B}(B^0_d \to \pi^+\pi^-)}{\mathcal{B}(\bar{B}^0_d \to \pi^+\pi^-) + \mathcal{B}(B^0_d \to \pi^+\pi^-)} \; , \\ \mathcal{A}_{00} &\equiv \frac{\mathcal{B}(\bar{B}^0_d \to \pi^0\pi^0) - \mathcal{B}(B^0_d \to \pi^0\pi^0)}{\mathcal{B}(\bar{B}^0_d \to \pi^0\pi^0) + \mathcal{B}(B^0_d \to \pi^0\pi^0)} \; , \\ \mathcal{A}_{0\pm} &\equiv \frac{\mathcal{B}(B^-_u \to \pi^0\pi^-) - \mathcal{B}(B^+_u \to \pi^0\pi^+)}{\mathcal{B}(B^-_u \to \pi^0\pi^-) + \mathcal{B}(B^+_u \to \pi^0\pi^+)} \; . \end{aligned} \tag{4}$$

Taking account of Eqs. (1)–(4), one can easily obtain the magnitude of each decay amplitude:

$$\begin{aligned} \left| A(\bar{B}^0_d \to \pi^+\pi^-) \right| &\propto \sqrt{\frac{\mathcal{B}_{+-}}{\tau_0} \left(1 + \mathcal{A}_{+-}\right)} \; , \\ \left| A(\bar{B}^0_d \to \pi^0\pi^0) \right| &\propto \sqrt{\frac{\mathcal{B}_{00}}{\tau_0} \left(1 + \mathcal{A}_{00}\right)} \; , \\ \left| A(B^-_u \to \pi^0\pi^-) \right| &\propto \sqrt{\frac{\mathcal{B}_{0\pm}}{\tau_\pm} \left(1 + \mathcal{A}_{0\pm}\right)} \; ; \end{aligned} \tag{5}$$

and

$$\begin{aligned} \left| A(B^0_d \to \pi^+\pi^-) \right| &\propto \sqrt{\frac{\mathcal{B}_{+-}}{\tau_0} \left(1 - \mathcal{A}_{+-}\right)} \; , \\ \left| A(B^0_d \to \pi^0\pi^0) \right| &\propto \sqrt{\frac{\mathcal{B}_{00}}{\tau_0} \left(1 - \mathcal{A}_{00}\right)} \; , \\ \left| A(B^+_u \to \pi^0\pi^+) \right| &\propto \sqrt{\frac{\mathcal{B}_{0\pm}}{\tau_\pm} \left(1 - \mathcal{A}_{0\pm}\right)} \; , \end{aligned} \tag{6}$$

where τ_0 and $\tau_\pm$ denote the lifetimes of neutral and charged B mesons, respectively. The present experimental data yield $\kappa \equiv \tau_0/\tau_\pm \approx 0.92$ [4]. In Eqs. (5) and (6), we have neglected the tiny phase space difference between $\pi^0\pi^\pm$ and $\pi^+\pi^-$ (or $\pi^0\pi^0$) states. It is obvious that the relevant decay amplitudes cannot be determined, unless the CP-violating asymmetries $\mathcal{A}_{+-}$, $\mathcal{A}_{00}$ and $\mathcal{A}_{0\pm}$ are all measured.

Under isospin symmetry and in the neglect of electroweak penguin contributions [9], the amplitudes of $\bar{B}^0_d \to \pi^+\pi^-$, $\bar{B}^0_d \to \pi^0\pi^0$ and $B^-_u \to \pi^0\pi^-$ decays form an isospin triangle in the complex plane:

$$A(\bar{B}^0_d \to \pi^+\pi^-) + \sqrt{2} A(\bar{B}^0_d \to \pi^0\pi^0) \; = \; \sqrt{2} A(B^-_u \to \pi^0\pi^-) \; . \tag{7}$$

Similarly,

$$A(B_d^0 \to \pi^+\pi^-) + \sqrt{2}A(B_d^0 \to \pi^0\pi^0) \;=\; \sqrt{2}A(B_u^+ \to \pi^0\pi^+) \; . \tag{8}$$

In this approximation, the magnitudes of $A(B_u^- \to \pi^0\pi^-)$ and $A(B_u^+ \to \pi^0\pi^+)$ are identical to each other [3]. Thus the CP-violating asymmetry $\mathcal{A}_{0\pm}$ vanishes. What we are concerned about is the relative phase between the amplitudes of $\bar{B}_d^0 \to \pi^+\pi^-$ (or $B_d^0 \to \pi^+\pi^-$) and $\bar{B}_d^0 \to \pi^0\pi^0$ (or $B_d^0 \to \pi^0\pi^0$) decays:

$$\begin{aligned} \phi &\equiv \arg\left[\frac{A(\bar{B}_d^0 \to \pi^+\pi^-)}{A(\bar{B}_d^0 \to \pi^0\pi^0)}\right] \; , \\ \varphi &\equiv \arg\left[\frac{A(B_d^0 \to \pi^+\pi^-)}{A(B_d^0 \to \pi^0\pi^0)}\right] \; . \end{aligned} \tag{9}$$

At the tree level, $\varphi = \phi$ holds. A difference between ϕ and φ measures the QCD penguin effects in $B_d \to \pi^+\pi^-$ and $B_d \to \pi^0\pi^0$ transitions. With the help of Eqs. (5)–(8), we find

$$\begin{aligned} \cos\phi &= \frac{2\kappa\mathcal{B}_{0\pm} - \mathcal{B}_{+-}\left(1 + \mathcal{A}_{+-}\right) - 2\mathcal{B}_{00}\left(1 + \mathcal{A}_{00}\right)}{2\sqrt{2\mathcal{B}_{+-}\mathcal{B}_{00}\left(1 + \mathcal{A}_{+-}\right)\left(1 + \mathcal{A}_{00}\right)}} \; , \\ \cos\varphi &= \frac{2\kappa\mathcal{B}_{0\pm} - \mathcal{B}_{+-}\left(1 - \mathcal{A}_{+-}\right) - 2\mathcal{B}_{00}\left(1 - \mathcal{A}_{00}\right)}{2\sqrt{2\mathcal{B}_{+-}\mathcal{B}_{00}\left(1 - \mathcal{A}_{+-}\right)\left(1 - \mathcal{A}_{00}\right)}} \; , \end{aligned} \tag{10}$$

where $\mathcal{A}_{0\pm} = 0$ has been taken into account. One can see that $\varphi = \phi$ would hold, if both $\mathcal{A}_{+-}$ and $\mathcal{A}_{00}$ were vanishing. Hence the difference between ϕ and φ results from the penguin-induced CP violation in $B_d \to \pi^+\pi^-$ and $B_d \to \pi^0\pi^0$ decays.

To get a ball-park feeling of the discrepancy between ϕ and φ, we make use of the QCD factorization to estimate the ratio of $A(\bar{B}_d^0 \to \pi^+\pi^-)$ to $A(\bar{B}_d^0 \to \pi^0\pi^0)$ and that of $A(B_d^0 \to \pi^+\pi^-)$ to $A(B_d^0 \to \pi^0\pi^0)$ [10]. The results are

$$\begin{aligned} \frac{A(\bar{B}_d^0 \to \pi^+\pi^-)}{A(\bar{B}_d^0 \to \pi^0\pi^0)} &= \frac{\left(a_1 + a_4^u + a_6^u\xi_\pi\right) R_b e^{-i\gamma} + \left(a_4^c + a_6^c\xi_\pi\right)}{\left(a_2 - a_4^u - a_6^u\xi_\pi\right) R_b e^{-i\gamma} - \left(a_4^c + a_6^c\xi_\pi\right)} \; , \\ \frac{A(B_d^0 \to \pi^+\pi^-)}{A(B_d^0 \to \pi^0\pi^0)} &= \frac{\left(a_1 + a_4^u + a_6^u\xi_\pi\right) R_b e^{+i\gamma} + \left(a_4^c + a_6^c\xi_\pi\right)}{\left(a_2 - a_4^u - a_6^u\xi_\pi\right) R_b e^{+i\gamma} - \left(a_4^c + a_6^c\xi_\pi\right)} \; , \end{aligned} \tag{11}$$

where $R_b \equiv |V_{ud}V_{ub}^*|/|V_{cd}V_{cb}^*|$ and $\gamma \equiv \arg[-(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)]$ are two parameters of the CKM unitarity triangle [11]; a_1, a_2, $a_4^{u,c}$ and $a_6^{u,c}$ denote the effective Wilson coefficients [12]; ξ_π is associated with the penguin operators $Q_{5,6}$ [13] and can be given as

$$\xi_\pi \;=\; \frac{2m_\pi^2}{(m_b - m_u)(m_u + m_d)} \tag{12}$$

under isospin symmetry. For illustration, we typically adopt $R_b \approx 0.37$ and $\gamma \approx 65^\circ$ [11] as well as $a_1 \approx 1.038 + 0.018i$, $a_2 \approx 0.082 - 0.080i$, $a_4^u \approx -0.029 - 0.015i$ and $a_4^c \approx -0.034 - 0.008i$ at the scale $\mu = m_b$ [12]. The formally power-suppressed QCD coefficients $a_6^{u,c}$ can be neglected in the heavy quark limit [12], because of $\xi_\pi \sim \mathcal{O}(1)$. Then we arrive at $A(\bar{B}_d^0 \to \pi^+\pi^-)/A(\bar{B}_d^0 \to \pi^0\pi^0) \approx 7.4e^{-i17^\circ}$ and $A(B_d^0 \to \pi^+\pi^-)/A(B_d^0 \to \pi^0\pi^0) \approx 4.3e^{+i44^\circ}$.

As a result, $\phi \approx -17^\circ$ and $\varphi \approx 44^\circ$ are obtained. The big difference $\varphi - \phi \approx 61^\circ$ implies the existence of significant QCD penguin effects and large direct CP violation in $B_d \rightarrow \pi\pi$ decays. It should be noted that ϕ and φ are neither pure weak phases nor pure strong phases, since the treel and penguin amplitudes of each $B_d \rightarrow \pi\pi$ decay mode involve different weak and strong phases. It should also be noted that the numerical results for ϕ and φ are just illustrative, because possible final-state rescattering effects have not been taken into account in Eq. (11).

Once the CP-violating asymmetries $\mathcal{A}_{+-}$ and $\mathcal{A}_{00}$ are measured at B factories, it will be possible to determine ϕ and φ from Eq. (10) independently of the specific QCD models. A hint of $\mathcal{A}_{+-} \neq 0$ (of $\sim 2\sigma$ significance) has recently been reported by the Belle Collaboration, but the BaBar Collaboration's preliminary result for $\mathcal{A}_{+-}$ is consistent with zero [2]. There is no experimental information about $\mathcal{A}_{00}$ at present. In this situation, we show the numerical correlation between ϕ and φ in Fig. 1(a) and that between $\mathcal{A}_{+-}$ and $\mathcal{A}_{00}$ in Fig. 1(b), where current data listed in Eqs. (1)–(3) have been used. The points in Fig. 1(a) are generated from scanning the error bars of $\mathcal{B}_{+-}$, $\mathcal{B}_{00}$ and $\mathcal{B}_{0\pm}$. The light region in Fig. 1(b) is generated in a similar way and corresponds to the allowed ranges of $\cos\phi$ and $\cos\varphi$ in Fig. 1(a), while the dark region in Fig. 1(b) results from inputting the central values of $\mathcal{B}_{+-}$, $\mathcal{B}_{00}$ and $\mathcal{B}_{0\pm}$. Some comments and discussions are in order.

(1) Current experimental results of $\mathcal{B}_{+-}$, $\mathcal{B}_{00}$ and $\mathcal{B}_{0\pm}$ can impose some limited but model-independent constraints on the values of (ϕ, φ) and $(\mathcal{A}_{+-}, \mathcal{A}_{00})$. For example, the possibilities $\cos\phi = \cos\varphi = 0$ and $|\mathcal{A}_{+-}| = |\mathcal{A}_{00}| = 1$ are not allowed. As a consequence of $|\mathcal{A}_{+-}| < 1$ and $|\mathcal{A}_{00}| < 1$, the regions for $\cos\phi < -0.2$ and $\cos\varphi < -02$ have completely been excluded by the present experimental data.

(2) The error bars of $\mathcal{B}_{+-}$, $\mathcal{B}_{00}$ and $\mathcal{B}_{0\pm}$ influence the parameter space of (ϕ, φ) or $(\mathcal{A}_{+-}, \mathcal{A}_{00})$ to a very limited extent. If $\mathcal{A}_{+-}$ is measured, one can get a roughly allowed range of $\mathcal{A}_{00}$ from Fig. 1(b). For instance, $\mathcal{A}_{+-} \geq 0.6$ and $\mathcal{A}_{00} \geq 0.6$ cannot simultaneously hold. Nor can $\mathcal{A}_{+-} \leq -0.6$ and $\mathcal{A}_{00} \leq -0.6$ simultaneously hold.

(3) As the tree-level amplitude of $B^0_d \rightarrow \pi^0\pi^0$ or $\bar{B}^0_d \rightarrow \pi^0\pi^0$ is color-suppressed, its magnitude might be comparable with the size of the corresponding penguin amplitude. Then $|\mathcal{A}_{00}|$ is expected to be larger than $|\mathcal{A}_{+-}|$ in the QCD factorization. However, the elastic rescattering effects in the final states of $B \rightarrow \pi\pi$ transitions could easily spoil this naive expectation. It is therefore useful to analyze the relevant experimental data in a model-independent way.

(4) The validity of our numerical results relies on the validity of the isospin relations in Eqs. (7) and (8), which hold in the assumption of negligible electroweak penguin effects. The electroweak penguin contributions to $B \rightarrow \pi\pi$ transitions are generally expected to be insignificant [9]. This expectation would be problematic or incorrect, if $\mathcal{A}_{0\pm} \neq 0$ were experimentally established. It is also worth mentioning that final-state interactions in $B \rightarrow \pi\pi$ decays include both elastic $\pi\pi \rightleftharpoons \pi\pi$ rescattering and some possible inelastic rescattering effects [2]. Whether inelastic final-state interactions are negligibly small or not remains an

[2]The amplitudes of $\bar{B}^0_d \rightarrow \pi^+\pi^-$, $\bar{B}^0_d \rightarrow \pi^0\pi^0$ and $B^-_u \rightarrow \pi^0\pi^-$ decay modes (or their CP-conjugate processes) may still form an isospin triangle in the complex plane, even if the inelastic $\pi\pi \rightleftharpoons D\bar{D}$

open question [16]. To answer this question requires more precise data on both the branching fractions and the CP-violating asymmetries of $B \rightarrow \pi\pi$ decays.

We proceed to discuss the extraction of the CP-violating phase α with the help of the isospin triangles in Eqs. (7) and (8). As shown in Ref. [3], α depends on two CP-violating observables which arise from the interplay of decay and B^0_d-$\bar{B}^0_d$ mixing in $B_d \rightarrow \pi^+\pi^-$ and $B_d \rightarrow \pi^0\pi^0$ decays:

$$\begin{aligned}\chi_{+-} &\equiv \mathrm{Im}\left[\frac{q}{p}\cdot\frac{A(\bar{B}^0_d \rightarrow \pi^+\pi^-)}{A(B^0_d \rightarrow \pi^+\pi^-)}\right] = |R|\sin[2(\alpha+\Theta)] \; , \\ \chi_{00} &\equiv \mathrm{Im}\left[\frac{q}{p}\cdot\frac{A(\bar{B}^0_d \rightarrow \pi^0\pi^0)}{A(B^0_d \rightarrow \pi^0\pi^0)}\right] = |\bar{R}|\sin[2(\alpha+\bar{\Theta})] \; , \end{aligned} \tag{13}$$

where

$$\begin{aligned} R = |R|e^{2i\Theta} &= \frac{1-\bar{r}}{1-r} \; , \\ \bar{R} = |\bar{R}|e^{2i\bar{\Theta}} &= \frac{2+\bar{r}}{2+r} \; , \end{aligned} \tag{14}$$

with $r = |r|e^{i\theta} \equiv A_0/A_2$ and $\bar{r} = |\bar{r}|e^{i\bar{\theta}} \equiv \bar{A}_0/\bar{A}_2$ being the ratios of $I = 0$ and $I = 2$ isospin amplitudes in $B^0_d \rightarrow \pi^+\pi^-$ (or $\pi^0\pi^0$) and $\bar{B}^0_d \rightarrow \pi^+\pi^-$ (or $\pi^0\pi^0$) decays. Following Ref. [17], we obtain

$$\begin{aligned} |r| &= \sqrt{3(a+b)-2} \; , \\ \theta &= \pm \arccos\left[\frac{6b-3a-2}{4\sqrt{3(a+b)-2}}\right] \; ; \end{aligned} \tag{15}$$

and

$$\begin{aligned} |\bar{r}| &= \sqrt{3(\bar{a}+\bar{b})-2} \; , \\ \bar{\theta} &= \pm \arccos\left[\frac{6\bar{b}-3\bar{a}-2}{4\sqrt{3(\bar{a}+\bar{b})-2}}\right] \; , \end{aligned} \tag{16}$$

where

$$\begin{aligned} a \equiv \left|\frac{A(B^0_d \rightarrow \pi^+\pi^-)}{A(B^+_u \rightarrow \pi^0\pi^+)}\right|^2 &= \frac{1}{\kappa}\cdot\frac{\mathcal{B}_{+-}}{\mathcal{B}_{0\pm}}\left(1-\mathcal{A}_{+-}\right) \; , \\ b \equiv \left|\frac{A(B^0_d \rightarrow \pi^0\pi^0)}{A(B^+_u \rightarrow \pi^0\pi^+)}\right|^2 &= \frac{1}{\kappa}\cdot\frac{\mathcal{B}_{00}}{\mathcal{B}_{0\pm}}\left(1-\mathcal{A}_{00}\right) \; ; \end{aligned} \tag{17}$$

---

rescattering effects are taken into account [14]. In this specific case, the analytical result obtained in Eq. (10) remains valid. It is even expected that inelastic final-state interactions could account for the existing discrepancy between experimental data and QCD models for $D \rightarrow \pi\pi$ and $B \rightarrow \pi\pi$ decays [15].

and

$$
\begin{aligned}
\bar{a} \equiv \left| \frac{A(\bar{B}_d^0 \to \pi^+\pi^-)}{A(B_u^- \to \pi^0\pi^-)} \right|^2 &= \frac{1}{\kappa} \cdot \frac{\mathcal{B}_{+-}}{\mathcal{B}_{0\pm}} \left(1 + \mathcal{A}_{+-}\right) \; , \\
\bar{b} \equiv \left| \frac{A(\bar{B}_d^0 \to \pi^0\pi^0)}{A(B_u^- \to \pi^0\pi^-)} \right|^2 &= \frac{1}{\kappa} \cdot \frac{\mathcal{B}_{00}}{\mathcal{B}_{0\pm}} \left(1 + \mathcal{A}_{00}\right) \; .
\end{aligned}
\tag{18}
$$

Clearly r (or $\bar{r}$) can be determined up to a twofold ambiguity in the sign of its phase, if both $\mathcal{A}_{+-}$ and $\mathcal{A}_{00}$ are measured. One can in turn determine R (or $\bar{R}$), both its magnitude and its phase. It is then possible to extract the weak phase α from Eq. (13) through the time-dependent measurement of χ_{+-} and (or) χ_{00} in $B_d \to \pi^+\pi^-$ and (or) $B_d \to \pi^0\pi^0$ decays. On the $\Upsilon(4S)$ resonance, the time-dependent rate asymmetry between $B_d^0(t)$ and $\bar{B}_d^0(t)$ decays into a CP eigenstate f reads as

$$
\begin{aligned}
\Delta_f(t) &\equiv \frac{\Gamma[\bar{B}_d^0(t) \to f] - \Gamma[B_d^0(t) \to f]}{\Gamma[\bar{B}_d^0(t) \to f] + \Gamma[B_d^0(t) \to f]} \\
&= \frac{|\rho_f|^2 - 1}{|\rho_f|^2 + 1} \cos(\Delta M_d \cdot t) \; + \frac{2\mathrm{Im}\left(\dfrac{q}{p}\rho_f\right)}{|\rho_f|^2 + 1} \sin(\Delta M_d \cdot t) \; ,
\end{aligned}
\tag{19}
$$

where $\rho_f \equiv A(\bar{B}_d^0 \to f)/A(B_d^0 \to f)$ and tiny CP violation in B_d^0-$\bar{B}_d^0$ mixing has been neglected (i.e., $|q/p| \approx 1$) [18]. For $f = \pi^+\pi^-$ and $\pi^0\pi^0$, we take account of Eqs. (4)–(6) and (13) and obtain

$$
\begin{aligned}
\Delta_{\pi^+\pi^-}(t) &= \mathcal{A}_{+-} \cos(\Delta M_d \cdot t) + (1 - \mathcal{A}_{+-})\chi_{+-} \sin(\Delta M_d \cdot t) \; , \\
\Delta_{\pi^0\pi^0}(t) &= \mathcal{A}_{00} \cos(\Delta M_d \cdot t) \; + \; (1 - \mathcal{A}_{00})\chi_{00} \sin(\Delta M_d \cdot t) \; .
\end{aligned}
\tag{20}
$$

Clearly both $\mathcal{A}_{+-}$ (or $\mathcal{A}_{00}$) and χ_{+-} (or χ_{00}) can be determined from measuring the time distribution of $\Delta_{\pi^+\pi^-}$ (or $\Delta_{\pi^0\pi^0}$) at asymmetric B factories. Compared with Refs. [3,17], the formulas obtained in Eqs. (13)–(18) and (20) establish a more *direct* relationship between α and the observables $\mathcal{B}_{+-}$, $\mathcal{B}_{00}$, $\mathcal{B}_{0\pm}$, $\mathcal{A}_{+-}$ and $\mathcal{A}_{00}$. Thus the usefulness of our results is noteworthy.

To summarize, we have presented a fresh isospin analysis of rare $B \to \pi\pi$ decays by taking account of the fact that the experimentally-reported branching fractions are charge-averaged and large CP violation may exist in them. We find that current experimental data can impose some useful and model-independent constraints on the parameter space of direct CP violation in $B_d \to \pi^+\pi^-$ and $B_d \to \pi^0\pi^0$ transitions. A straightforward relationship between the CP-violating phase α and the measurables of $B \to \pi\pi$ decays has also been established. More precise measurements of such charmless B decays will allow us to test the consistency of the Kobayashi-Maskawa mechanism of CP violation and probe possible new physics beyond the standard model.

The author likes to thank Y.F. Zhou for useful discussions and partial involvement at the very early stage of this work (in 2002). This research was supported in part by the National Nature Science Foundation of China.

REFERENCES

[1] BaBar Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **87**, 091801 (2001); Belle Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **87**, 091802 (2001).

[2] H. Yamamoto, plenary talk given at the International Europhysics Conference on High Energy Physics, July 2003, Aachen, Germany; T. Browder, talk given at the 21st International Symposium on Lepton and Photon Interactions at High Energies (LP 03), August 2003, Batavia, Illinois, USA; H. Jawahery, talk given at the same conference (LP 03).

[3] M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990); Y. Grossman and H.R. Quinn, Phys. Rev. D **58**, 017504 (1998).

[4] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).

[5] BaBar Collaboration, B. Aubert *et al.*, hep-ex/0308012.

[6] Belle Collaboration, K. Abe *et al.*, hep-ex/0308040.

[7] For recent reviews with extensive references, see: H.N. Li, hep-ph/0303116; M. Beneke and M. Neubert, hep-ph/0308039.

[8] J. Fry, talk given at the 21st International Symposium on Lepton and Photon Interactions at High Energies (LP 03), August 2003, Batavia, Illinois, USA.

[9] N.G. Deshpande and X.G. He, Phys. Rev. Lett. **74**, 26 (1995); M. Gronau, O.F. Hernández, D. London, and J.L. Rosner, Phys. Rev. D **52**, 6374 (1995); R. Fleischer, Phys. Rept. **370**, 537 (2002).

[10] Z.Z. Xing, Phys. Lett. B **493**, 301 (2000).

[11] A.J. Buras, hep-ph/0307203.

[12] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. B **591**, 313 (2000).

[13] G. Buchalla, A.J. Buras, and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).

[14] C. Hamzaoui and Z.Z. Xing, Phys. Lett. B **360**, 131 (1995).

[15] H.Y. Cheng, Eur. Phys. J. C **26**, 551 (2003).

[16] J.F. Donoghue, E. Golowich, A.A. Petrov, and J.M. Soares, Phys. Rev. Lett. **77**, 2178 (1996); Z.Z. Xing, Phys. Rev. D **53**, 2847 (1996).

[17] D. Du and Z.Z. Xing, Phys. Rev. D **47**, 2825 (1993).

[18] See, e.g., Z.Z. Xing, Phys. Rev. D **53**, 204 (1996).

FIGURES

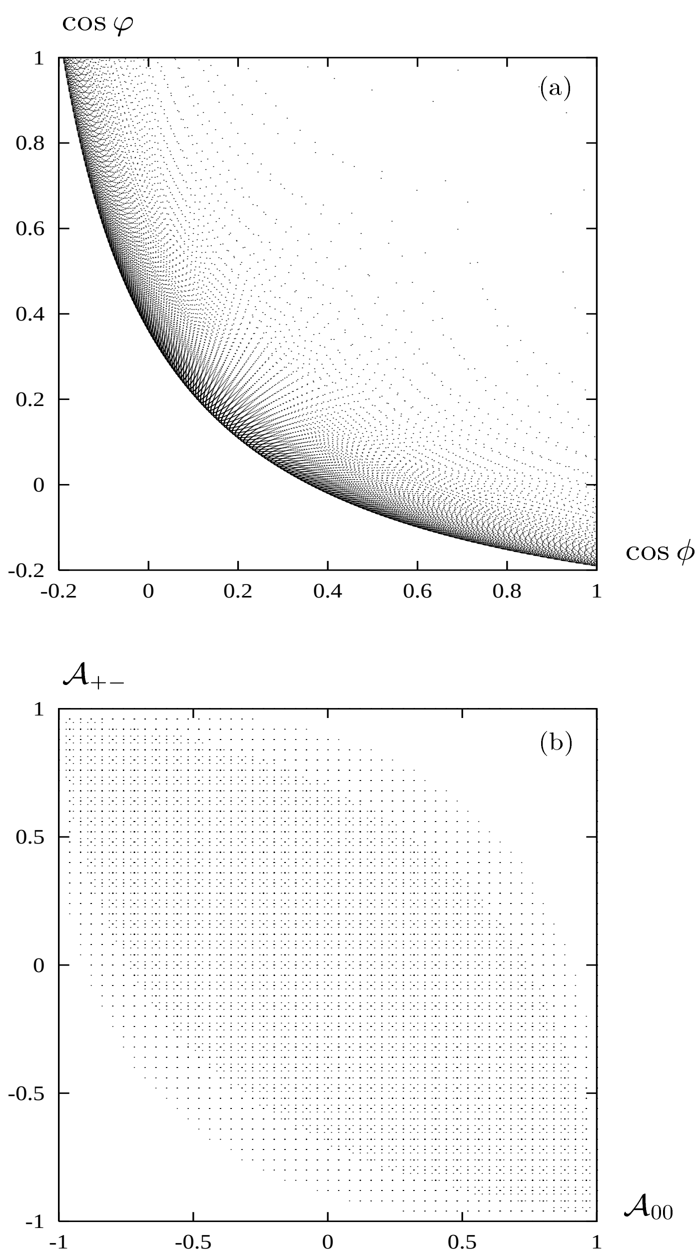

FIG. 1. Constraints of current experimental data on (a) the parameter space of $\cos\phi$ and $\cos\varphi$ and (b) the parameter space of $\mathcal{A}_{00}$ and $\mathcal{A}_{+-}$. Note that the dark region of (b) corresponds to the central values of $\mathcal{B}_{+-}$, $\mathcal{B}_{00}$ and $\mathcal{B}_{0\pm}$ given in Eqs. (1)–(3).